2026 HSC Mathematics Advanced
Solutions

Question 1-10

1. **C**.

   Leading coefficient must be positive since as $x \to \infty$, $y \to \infty$. Therefore, not **A** or **B**. x-intercepts at -4, 1 and 3 gives option **C**.

2. **D**.

   Recall that the mode of a continuous random variable X corresponds to the value of x that maximises $f(x)$ where $f(x)$ is the probability density function of X. Hence we see visually that the mode is at $2\sqrt{2}$ (and **not** $\sqrt{2}/2$ which is the corresponding value of $f(x)$!).

3. **B**.

   Observe that the original graph must have been horizontally dilated by a factor of 2 and also reflected about the x-axis. So we expect x to be replaced with $x/2$ as well as y to be replaced with $-y$. This corresponds to **B**.

4. **D**.

   A is incorrect since a correlation of 0.62 does not mean that 62% of students achieve higher scores. **B** is incorrect since correlation does not imply causation. **C** is wrong because, again, the correlation of 0.62 does not mean 62% of students with higher scores study more.

5. **A**.

   From the diagram, the maximum height occurs at $t = 3$. So we substitute to find the value $h(3) = 45 + 12(3) - 2(3)^2 = 63$. However the object was already initially at $h(0) = 45$. So it has risen $63 - 45 = 18$ metres which corresponds to (A).

6. **A**.

$$\begin{aligned}
\frac{\sin \angle EGF}{81} &= \frac{\sin 35^\circ}{50} \\
\angle EGF &= 180^\circ - \sin^{-1}\left(\frac{81 \times \sin 35^\circ}{50}\right) \quad \text{(as } \angle EGF \text{ is obtuse)} \\
&= 180^\circ - 68.3^\circ = 111.7^\circ \\
\theta &= 111.7^\circ - 55^\circ = 56.7^\circ
\end{aligned}$$

7. **B**.

   The sum is -3 only when one roll is -4 and the other is 1, or one roll is -3 and the other is 0. Each pair can occur in either order, and the rolls are independent, so

$$\begin{aligned}
P(\text{sum} = -3) &= 2\,P(X = -4)\,P(X = 1) + 2\,P(X = -3)\,P(X = 0) \\
&= 2 \cdot \frac{3}{18} \cdot \frac{1}{18} + 2 \cdot \frac{4}{18} \cdot \frac{3}{18} \\
&= \frac{6}{324} + \frac{24}{324} \\
&= \frac{30}{324} = \frac{5}{54}.
\end{aligned}$$

Vertex Mathematics

8. **C**.

Since f is odd,

$$\begin{aligned} Q &= \int_{-a}^{a} f(x)\,dx = 0, \\ S &= \int_{-a}^{0} f(x)\,dx = -\int_{0}^{a} f(x)\,dx = -P < 0, \\ R &= \int_{-a}^{1} f(x)\,dx = -\int_{1}^{a} f(x)\,dx > 0. \end{aligned}$$

To compare P and R, which are both clearly positive, consider

$$\begin{aligned} P - R &= \int_{0}^{a} f(x)\,dx + \int_{1}^{a} f(x)\,dx \\ &= \int_{0}^{1} f(x)\,dx + 2\int_{1}^{a} f(x)\,dx \\ &> 0 \qquad \text{(from the graph)}. \end{aligned}$$

Hence, $S < Q < R < P$, giving option **C**.

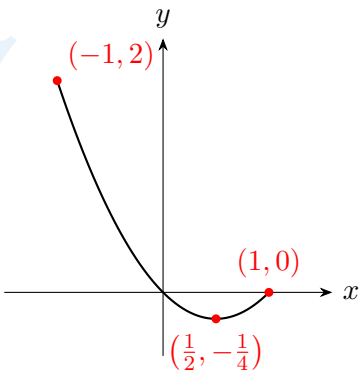
9. **A**.

The shaded probability is $P(Z < -1.5) + P(Z > 1.2)$. By the symmetry of the normal curve, $P(Z > 1.2) = P(Z < -1.2)$, so

$$\begin{aligned} P(Z < -1.5) + P(Z > 1.2) &= P(Z < -1.5) + P(Z < -1.2) \\ &= P(Z < -1.5) + \big[P(Z < -1.5) + P(-1.5 < Z < -1.2)\big] \\ &= 2P(Z < -1.5) + P(-1.5 < Z < -1.2). \end{aligned}$$

10. **D**.

The range of $g(x) = \sin x$ is $[-1, 1]$. Drawing a graph of $f(x) = x^2 - x$ shows that on this domain, the range of $f(x)$ is $[-1/4, 2]$ so if $f(g(x)) = k$ has a solution, $k \in [-1/4, 2]$.



Question 11

We have $y' = 3x^2 + 3$ which equals $3(-1)^2 + 3 = 6$ at $x = -1$. Hence the gradient of the tangent at $x = -1$ is 6. **2**

Vertex Mathematics

Question 12

(a) There are 13 observations in total so the median will be the middle observation (the 7th) of 22. Hence Q_1 is the median of the first 6 observations which is $Q_1 = \dfrac{15+17}{2} = 16$ and Q_3 is the median of the last 6 observations which is $Q_3 = \dfrac{27+28}{2} = 27.5$. Therefore **2**

$$\text{IQR} = 27.5 - 16 = 11.5.$$

(b) We would classify an outlier to the right of Q_3 as any value that exceeds $Q_3 + 1.5 \times \text{IQR} = 27.5 + 1.5(11.5) = 44.75$. Since m is the number of cars, it must be a whole number. Hence the greatest value of m is 44. **2**

Question 13

(a) Let T_n be the number of seats in the nth row. So we have that T_n is of the form $a + (n-1)d$. Substituting $n = 3$ and $n = 6$ we have the simultaneous equations: **2**

$$a + 2d = 30 \qquad (1)$$
$$a + 5d = 42 \qquad (2).$$

Subtracting (1) from (2) gives $3d = 12$ so $d = 4$. Substituting $d = 4$ back into the first equation gives $a = 30 - 2 \times 4 = 22$. Hence there are 22 seats in the first row.

(b) We use the formula $S_n = \dfrac{n}{2}(2a + (n-1)d)$ and substitute $a = 22$, $d = 4$ and $n = 9$ to obtain $S_9 = \dfrac{9}{2}[2(22) + 8(4)] = 342$ seats. **2**

Question 14

The amplitude is 3 and the centre is 1 which gives the range of $h(x) \in [1-3, 1+3] = [-2, 4]$. The period is given by $T = \dfrac{2\pi}{2} = \pi$. **2**

Question 15

We use the formula $\dfrac{d}{dx}\left(a^{f(x)}\right) = \ln(a) f'(x) a^{f(x)}$ to obtain **2**

$$\frac{dy}{dx} = \frac{d}{dx}\left(x^2+3\right)\ln(4)4^{x^2+3} = 2x\ln(4)4^{x^2+3}$$

(Note $2\ln 4$ can be alternatively written as $4\ln 2$ or $\ln 16$, it is not clear how students should have left their answer here though we expect all to recieve credit).

Vertex Mathematics

Question 16

(a) The point with the smallest y-value is between 6.05 and 6.125 so we know that it will be 6.1 when correct to one decimal place. Alternatively, the point is a tiny bit less than a quarter of the way between 6 and 6.5, so it is a little less than 6.125. **1**

(b) For every unit increase in the critics', there is an average 0.54 increase in the public's score. **2**

(c) This point is currently below the regression line. As a result we would expect the gradient of the new updated regression line to slightly increase to accommodate this new point (but it would not change drastically). **1**

Question 17

(a) At $t = 0$, $40 = ka^0 = k$, so $k = 40$. At $t = 1$, $32 = 40a$ so $a = 0.8$. So we have $a = 0.8$ and $k = 40$. **2**

(b) The battery retains $a \times 100\% = 80\%$ of its maximum storage capacity from the preceding year. In other words, the storage decreases by 20% each year as it gets more used. **1**

Question 18

(a) Solve for the intersection by setting each curve to be equal: **2**

$$\begin{aligned} 7 - 3x &= -x^2 + x + 12 \\ x^2 - 4x - 5 &= 0 \\ (x+1)(x-5) &= 0 \end{aligned}$$

so the x-coordinates of the points of intersection at $x = -1$ and $x = 5$.

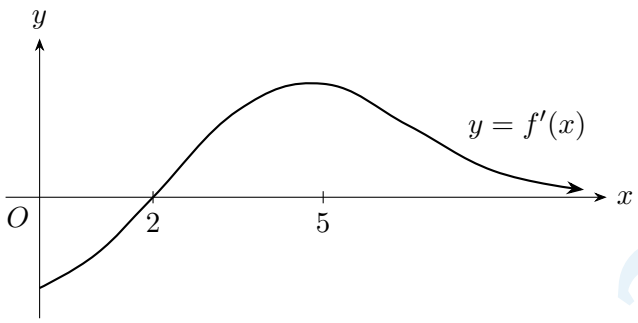
(b) From the supplied diagram, the quadratic function is above the linear function between $x = -1$ and $x = 5$. Hence the area is obtained by the integral **2**

$$\begin{aligned} A &= \int_{-1}^{5} \left[(-x^2 + x + 12) - (7 - 3x)\right] dx \\ &= \int_{-1}^{5} (-x^2 + 4x + 5)\, dx \\ &= \left[-\frac{x^3}{3} + 2x^2 + 5x\right]_{-1}^{5} \\ &= 36 \text{ units}^2. \end{aligned}$$

Vertex Mathematics

Question 19

At $x = 2$, the graph obtains a stationary point so $f'(2) = 0$, and the point of inflection at $x = 5$ implies that the slope goes from increasing to decreasing. Hence $f'(5)$ is a maximum. As $x \to \infty$, the slope of $f(x)$ flattens out and so $f'(x) \to 0$, from the positive side. **3**



Question 20

(a) Following the "Walk" then "Not correct uniform" branches of the tree, **1**

$$P(\text{walk and not correct uniform}) = \frac{3}{10} \times \frac{1}{10} = \frac{3}{100}.$$

(b) Let E be the event that a student walks to school and is not wearing the correct uniform. From (a), $P(E) = \frac{3}{100}$, so $P(\overline{E}) = \frac{97}{100}$. Treating the two students as independent, **2**

$$\begin{aligned} P(\text{at least one student satisfies } E) &= 1 - P(\text{neither satisfies } E) \\ &= 1 - \left(\frac{97}{100}\right)^2 \\ &= 1 - \frac{9409}{10000} \\ &= \frac{591}{10000}. \end{aligned}$$

Question 21

(a) Using the quotient rule, **2**

$$g'(x) = \frac{(\ln x)(1) - x\left(\frac{1}{x}\right)}{(\ln x)^2} = \frac{\ln x - 1}{(\ln x)^2}.$$

(b) The stationary point occurs when $g'(x) = 0$, so $x = e$ (which is still in the domain $[2, 5]$). Since $g(x)$ is continuous on $x \in [2, 5]$, the global maximum will either occur at a stationary point or an endpoint. Se we compare the values of $g(2)$, $g(e)$ and $g(5)$: **3**

$$g(2) = \frac{2}{\ln 2}, \qquad g(e) = e, \qquad g(5) = \frac{5}{\ln 5}.$$

The largest of these three values is $g(5)$. Hence the global maximum on $[2, 5]$ is $\dfrac{5}{\ln 5}$.

Vertex Mathematics

Question 22

1

(a)

$$\begin{aligned}\frac{x+2}{x+1} &= \frac{x+1+1}{x+1} \\ &= \frac{x+1}{x+1} + \frac{1}{x+1} \\ &= 1 + \frac{1}{x+1}, \qquad x \neq -1\end{aligned}$$

(b) The graph of $y = \dfrac{1}{x}$ is translated 1 unit to the left and then 1 unit upwards. Note that the order in which these are done does not matter since they are transformations on different variables. **2**

Question 23

We know that $80 = \dfrac{1}{2}OB^2 \sin(45°)$, so $OB = \sqrt{160\sqrt{2}}$. Now, $\angle COB = 152°$ using the bearings. Using the cosine rule, **4**

$$BC^2 = OB^2 + OC^2 - 2 \cdot OB \cdot OC \cos(\angle COB).$$

Rounding to the nearest kilometre, we obtain $BC = 39$km.

Question 24

(a) Since 75% of the cobalt-60 has decayed, 25% remains, so $A = 250$ when $t = 10.54$: **2**

$$\begin{aligned}250 &= 1000e^{10.54k} \\ e^{10.54k} &= \frac{1}{4} \\ 10.54k &= \ln\left(\frac{1}{4}\right) = -\ln 4 \\ k &= -\frac{\ln 4}{10.54} = -0.131527\ldots \\ &\approx -0.1315 \text{ (to 4 decimal places).}\end{aligned}$$

(b) Differentiating, $\dfrac{dA}{dt} = 1000k\, e^{kt}$. Setting this equal to -9.48 with $k = -0.1315$, **2**

$$\begin{aligned}-131.5\, e^{-0.1315t} &= -9.48 \\ e^{-0.1315t} &= \frac{9.48}{131.5} \\ -0.1315t &= \ln\left(\frac{9.48}{131.5}\right) \\ t &= -\frac{1}{0.1315}\ln\left(\frac{9.48}{131.5}\right) \\ &= 19.998\ldots \\ &\approx 20 \text{ years.}\end{aligned}$$

(Using the unrounded value of k gives $t \approx 19.996$, which also rounds to 20.)

Vertex Mathematics

Question 25

Let X denote the weight of a watermelon. Then **2**

$$\begin{aligned} P(X < 3.75) &= 0.116 \\ P\left(Z < \frac{3.75 - 4.82}{\sigma}\right) &= 0.116 \\ P\left(Z < \frac{-1.07}{\sigma}\right) &= 0.116 \\ P\left(Z > \frac{1.07}{\sigma}\right) &= 0.116 \\ \therefore P(4.82 < X < 5.89) &= P\left(0 < Z < \frac{1.07}{\sigma}\right) \\ &= 0.5 - 0.116 \\ &= 0.384 \end{aligned}$$

Question 26

(a) Since the velocity crosses the t-axis at $t = 0.5$, the particle changes direction here. **1**

(b) We have **2**

$$\begin{aligned} a(t) &= v'(t) \\ &= \pi \sin(\pi t) e^{\cos(\pi t)}. \end{aligned}$$

So the acceleration at $t = 0.5$ is π ms^{-2}.

(c) For 4 sub-intervals, $h = 0.25$. Since distance requires speed instead of velocity, we have that **2**

$$d \approx \frac{0.25}{2}\left(|-1.72| + 2|-1.03| + 2 \cdot 0 + 2 \cdot 0.51 + 0.63\right).$$

Calculating, the total distance is $d = 0.68$m.

Question 27

The half-yearly interest rate is $i = 0.01$. **4**

Suppose Maya contributes \$750 for all 10 periods, then adds an extra \$750 for the final 8 periods. Using the annuity table,

$$FV = 750(10.4622) + 750(8.2857) = \$14060.93.$$

This exceeds \$14000, so she would need to double her contributions later, decreasing the FV, to fall below \$14000.

Doubling one period later loses the FV of one \$750 investment for 7 periods, so the new future value would be

$$FV - 750(1+i)^7 = 14060.91 - 750(1.01)^7 = 13256.81,$$

which is less than \$14000. Thus, the maximum is 2 contributions of \$750 before she will need to double her payments.

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Question 28

First note that for the cubic to have three $x-$intercepts, it must have two turning points. Furthermore, these turning points must have $y-$values which differ in sign. **4**

To find stationary points, we let the first derivative equal to zero.

$$\begin{aligned} y' = 12x^2 - 6x - 6 &= 0, \\ 6(2x+1)(x-1) &= 0, \\ x &= 1, -\frac{1}{2} \end{aligned}$$

To determine the nature of these stationary points, we use the second derivative, $y'' = 24x - 6$.

$$\begin{aligned} &\text{When } x = -\tfrac{1}{2},\ y'' = -18 < 0 \implies \text{maximum turning point at } \left(-\tfrac{1}{2},\ d + \tfrac{7}{4}\right) \\ &\text{When } x = 1,\ y'' = 18 > 0 \implies \text{minimum turning point at } (1,\ d-5) \end{aligned}$$

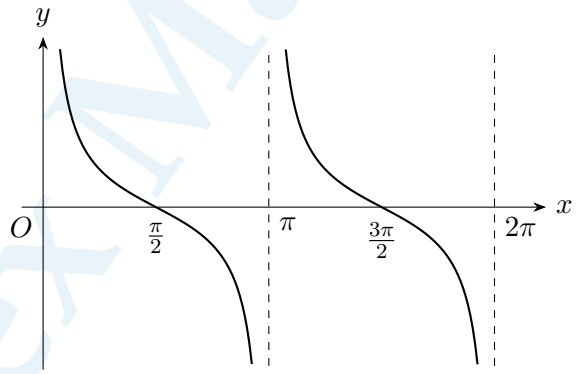
Now, we need the function value of the minimum turning point to be negative, and the function value of the maximum turning point to be positive.

$$\begin{aligned} d + \frac{7}{4} > 0 &\implies d > -\frac{7}{4} \\ d - 5 < 0 &\implies d < 5 \end{aligned}$$

Hence, our final solution is $-\dfrac{7}{4} < d < 5$.

Question 29

(a) **2**



(b) Notice $\ln(\cot x)$ is defined only when $\cot x > 0$. From the graph in (a), this occurs when **2**

$$0 < x < \frac{\pi}{2} \qquad \text{or} \qquad \pi < x < \frac{3\pi}{2}.$$

The endpoints are excluded since $\cot x$ is undefined at $x = 0, \pi$ and equals 0 at $x = \frac{\pi}{2}, \frac{3\pi}{2}$.

(c) Write $\cot x = \dfrac{\cos x}{\sin x}$. By the quotient rule, **2**

$$\frac{d}{dx}(\cot x) = \frac{\sin x(-\sin x) - \cos x \cos x}{\sin^2 x} = -\frac{\sin^2 x + \cos^2 x}{\sin^2 x} = -\frac{1}{\sin^2 x}.$$

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Hence, by the chain rule,

$$\begin{aligned}\frac{d}{dx}(\ln(\cot x)) &= \frac{1}{\cot x}\cdot\frac{d}{dx}(\cot x)\\ &= \frac{\sin x}{\cos x}\cdot\left(-\frac{1}{\sin^2 x}\right)\\ &= -\frac{1}{\sin x\cos x}\\ &= -\operatorname{cosec} x\sec x.\end{aligned}$$

Remark. It is tempting to write $\ln(\cot x)=\ln(\cos x)-\ln(\sin x)$ and differentiate each term. However, this identity requires $\cos x>0$ and $\sin x>0$, which fails for $\pi<x<\frac{3\pi}{2}$: there both $\cos x$ and $\sin x$ are negative, so $\ln(\cos x)$ and $\ln(\sin x)$ are undefined, even though $\ln(\cot x)$ is defined. Since this is a proof question, we instead differentiated $\cot x$ directly, which makes the result valid on the entire domain found in (b). (Splitting with absolute values, $\ln|\cos x|-\ln|\sin x|$, would also be valid.)

Question 30

(a) In the right-angled triangle MAO, the height of the pyramid is $MO=\ell\sin\theta$ and $AO=\ell\cos\theta$. Since O is the centre of the square, AO is half a diagonal, so $AC=2\ell\cos\theta$. A square with diagonal d has area $\frac{d^2}{2}$, so **3**

$$\text{Area of } ABCD=\frac{(2\ell\cos\theta)^2}{2}=2\ell^2\cos^2\theta.$$

Hence

$$\begin{aligned}V &= \frac{1}{3}\times\text{base area}\times\text{height}\\ &= \frac{1}{3}\left(2\ell^2\cos^2\theta\right)(\ell\sin\theta)\\ &= \frac{2\ell^3}{3}\sin\theta\cos^2\theta.\end{aligned}$$

(b) Treat ℓ as fixed, with $0<\theta<\frac{\pi}{2}$. By the product rule, **4**

$$\begin{aligned}\frac{dV}{d\theta} &= \frac{2\ell^3}{3}\left(\cos\theta\cos^2\theta+\sin\theta\cdot 2\cos\theta(-\sin\theta)\right)\\ &= \frac{2\ell^3}{3}\cos\theta\left(\cos^2\theta-2\sin^2\theta\right)\\ &= \frac{2\ell^3}{3}\cos^3\theta\left(1-2\tan^2\theta\right).\end{aligned}$$

Since $\cos\theta>0$ on this interval, $\dfrac{dV}{d\theta}=0$ when

$$\tan^2\theta=\frac{1}{2},\quad\text{so}\quad\tan\theta=\frac{1}{\sqrt{2}}\ (\text{as }\theta\text{ is acute}).$$

Thus $\theta=\tan^{-1}\left(\frac{1}{\sqrt{2}}\right)\approx 0.6155$.

To confirm this is a maximum, note that $\tan\theta$ is increasing on $\left(0,\frac{\pi}{2}\right)$, so $1-2\tan^2\theta$ changes from positive to negative as θ passes through this value. Hence $\dfrac{dV}{d\theta}$ changes from positive to negative, and V has a maximum there.

Therefore the volume is a maximum when $\theta\approx 0.62$ radians (to 2 decimal places).

Vertex Mathematics

Question 31

(a) $\int \cos 2x \, dx = \frac{1}{2}\sin 2x + C_1$. **1**

(b) The integral of $f'(x)/f(x)$ is $\ln|f(x)| + C_2$. Substituting this along with (a), we have **3**

$$\begin{aligned}\ln|f(x)| + C_2 &= \frac{1}{2}\sin 2x + C_1, \\ \ln|f(x)| &= \frac{1}{2}\sin 2x + C, \text{ where } C = C_1 - C_2.\end{aligned}$$

We are given $f(0) = 1$, so substitute $x = 0$,

$$\ln|1| = \frac{1}{2}\sin 0 + C, \qquad \therefore C = 0.$$

So $\ln|f(x)| = \frac{1}{2}\sin 2x$. Finally, since we are told $f(x) > 0$ for all x, we can remove the absolute values and exponentiate both sides, to get

$$f(x) = e^{\frac{1}{2}\sin 2x}.$$

Question 32

(a) By the fundamental theorem of calculus, $g(x) = F(1) - F(x^3)$ where $F(t)$ is a primitive of $\sqrt{\tan(t)}$. **1**

(b) Differentiating, $g'(x) = \frac{d}{dx}(F(1) - F(x^3))$. Since $F(1)$ is a constant, its derivative is zero. On the other hand, by the chain rule, **2**

$$\begin{aligned}\frac{d}{dx}F(x^3) &= F'(x^3) \cdot \frac{d}{dx}(x^3) \\ &= 3x^2\sqrt{\tan x^3},\end{aligned}$$

where $F'(x) = f(x) = \sqrt{\tan x}$ by the other fundamental theorem of calculus.
Hence, $g'(x) = -3x^2\sqrt{\tan x^3}$.

Question 33

(a) First, note that the monthly interest rate is $\frac{0.06}{12} = 0.005$. **2**

$$\begin{aligned}A_1 &= 83000(1.005) - 760 \\ A_2 &= 1.005A_1 - 760(1.1) \\ &= 83000(1.005)^2 - 760(1.005 + 1.1) \\ A_3 &= 1.005A_2 - 760(1.1)^2 \\ &= 83000(1.005)^3 - 760\left[(1.005)^2 + (1.005)(1.1) + (1.1)^2\right],\end{aligned}$$

as required.

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(b) Generalising from part (a), **3**

$$A_n = 83000(1.005)^n - 760\left[(1.005)^{n-1} + (1.005)^{n-2}(1.1) + \cdots + (1.1)^{n-1}\right].$$

The expression in square brackets is a geometric series with first term $(1.005)^{n-1}$ and common ratio $\dfrac{1.1}{1.005}$.
Hence,

$$\begin{aligned} A_n &= 83000(1.005)^n - 760(1.005)^{n-1}\left[\frac{\left(\frac{1.1}{1.005}\right)^n - 1}{\frac{1.1}{1.005} - 1}\right] \\ &= 83000(1.005)^n - 760\left[\frac{(1.1)^n - (1.005)^n}{0.095}\right] \\ &= 83000(1.005)^n - 8000\left[(1.1)^n - (1.005)^n\right] \\ &= 91000(1.005)^n - 8000(1.1)^n, \end{aligned}$$

as required.

(c) The loan is fully repaid when $A_n \leq 0$. **3**

$$\begin{aligned} 91000(1.005)^n - 8000(1.1)^n &\leq 0 \\ \left(\frac{1.1}{1.005}\right)^n &\geq \frac{91000}{8000} \\ n &\geq \frac{\log\left(\frac{91000}{8000}\right)}{\log\left(\frac{1.1}{1.005}\right)} \\ n &\geq 26.919\ldots \end{aligned}$$

Thus, the final repayment occurs at the end of the 27th month.
The amount owing after 26 repayments is

$$\begin{aligned} A_{26} &= 91000(1.005)^{26} - 8000(1.1)^{26} \\ &\approx 8254.41. \end{aligned}$$

Since interest is added before the final repayment, the final payment is

$$\begin{aligned} 1.005A_{26} &= 1.005\left[91000(1.005)^{26} - 8000(1.1)^{26}\right] \\ &\approx \$8295.68. \end{aligned}$$

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